AP Calculus AB
Implicit Differentiation Notes

Name Key

# Implicit Differentiation

Find the derivative of $x^2 + y^2 = 10$

$y^2 = 10 - x^2$

$y = \pm\sqrt{10 - x^2}$

$y = \pm (10 - x^2)^{1/2}$

$y' = \pm\frac{1}{2}(10 - x^2)^{-1/2}(-2x)$

$y' = \pm\frac{x}{(10 - x^2)^{-1/2}}$

Now find the derivative using implicit differentiation

$x^2 + y^2 = 10$

$2x + 2y\frac{dy}{dx} = 0$

$2y\frac{dy}{dx} = -2x$

$\frac{dy}{dx} = -\frac{2x}{2y}$

$\frac{dy}{dx} = -\frac{x}{y}$

Try these:

1. $y^3 + y^2 - 5y - x^2 = -4$

$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} - 2x = 0$

$3y^2\frac{dy}{dx} + 2y\frac{dy}{dx} - 5\frac{dy}{dx} = 2x$

$\frac{dy}{dx}(3y^2 + 2y - 5) = 2x$

$\frac{dy}{dx} = \frac{2x}{3y^2 + 2y - 5}$

2. $1 = x^2y - xy^3$

$0 = \left[(2x)y + (x^2)\frac{dy}{dx}\right] - \left[(1)y^3 + (x)3y^2\frac{dy}{dx}\right]$

$0 = 2xy + x^2\frac{dy}{dx} - y^3 - 3xy^2\frac{dy}{dx}$

$3xy^2\frac{dy}{dx} - x^2\frac{dy}{dx} = 2xy - y^3$

$\frac{dy}{dx} = \frac{2xy - y^3}{3xy^2 - x^2}$

3. $x^3 - 2x^2y + 3xy^2 = 38$

$3x^2 - \left[(4x)y + (2x^2)\frac{dy}{dx}\right] + \left[(3)y^2 + (3x)2y\frac{dy}{dx}\right] = 0$

$3x^2 - 4xy - 2x^2\frac{dy}{dx} + 3y^2 + 6xy\frac{dy}{dx} = 0$

$3x^2 - 4xy + 3y^2 = 2x^2\frac{dy}{dx} - 6xy\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{3x^2 - 4xy + 3y^2}{2x^2 - 6xy}$

4. $\sin(x) + \cos(2y) = 1$

$\cos x + (-\sin(2y))\left(2\frac{dy}{dx}\right) = 0$

$\cos x - 2\sin(2y)\frac{dy}{dx} = 0$

$\cos x = 2\sin(2y)\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{\cos x}{2\sin(2y)}$

5. $y^2 = \frac{x^2 - 9}{x^2 + 9}$

$2y\frac{dy}{dx} = \frac{(2x)(x^2+9) - (x^2-9)(2x)}{(x^2+9)^2}$

$2y\frac{dy}{dx} = \frac{2x^3 + 18x - 2x^3 + 18x}{(x^2+9)^2}$

$\frac{dy}{dx} = \frac{18x}{2y(x^2+9)^2} = \frac{9x}{y(x^2+9)^2}$

6. $y = \sin(xy)$

$\frac{dy}{dx} = \cos(xy)\left[(1)y + (x)\frac{dy}{dx}\right]$

$\frac{dy}{dx} = y\cos(xy) + x\cos(xy)\frac{dy}{dx}$

$\frac{dy}{dx} - x\cos(xy)\frac{dy}{dx} = y\cos(xy)$

$\frac{dy}{dx} = \frac{y\cos(xy)}{1 - x\cos(xy)}$